Math141_Homework

Name: UID: Section #:

Instructions

- The problems indicated in green are to be turned in via Gradescope and are graded for completion. All other problems are suggested. It is recommended to complete **all problems** as this document is where Quiz questions will be pulled from.
- You will turn in the green problems and the Discussion Worksheet problems (which are graded for accuracy) to the same Gradescope assignment to count as your Homework grade for the week.

Calculus I Review

1) $\int \left(x^{3/5} - 8x^{5/3}\right) dx$

2) $\int_1^2 \frac{x^2+2x+3}{x} dx$

3) $\int \frac{(\ln x)^2}{x} dx$

4) $\int \frac{x}{\sqrt{x+3}} dx$

5) The graphs of $f(x) = x^2 + 1$ and $g(x) = 2x + 9$ enclose a region. Determine the area A of that region.

Solutions

1) $\frac{5}{8}x^{8/5} - 3x^{8/3} + C$
2) $\frac{7}{2} + 3\ln 2$
3) $\frac{1}{3}(\ln x)^3 + C$
4) $\frac{2}{3}(x+3)^{3/2} - 6(x+3)^{1/2} + C$
5) 36

Volume (6.1)

6) Let R be the region between the graph of $f(x) = \sec x$ and the x-axis on the interval $[-\pi/4, 0]$. Find the volume V of the solid obtained by revolving R about the x axis.

7) Let R be the region between the graphs of f and g on the given interval. Find the volume V of the solid obtained by revolving R about the x axis.

   a) $f(x) = x + 1, g(x) = x - 1; [1, 4]$

   b) $f(x) = \cos x + \sin x, g(x) = \cos x - \sin x; [0, \pi/4]$

8) Let $f(y) = \sqrt{1 + y^3}$ on $[1, 2]$. Let R be the region between the graph of f and the y axis on the given interval. Find the volume V of the solid obtained by revolving R about the y axis.

9) The region bounded by the curves $y = x^2 - 2x$ and $y = 3x$ is revolved about the line $y = -1$. Find the volume of the resulting solid.

10) Find the volume V of the solid that has a circular base with radius 1, and the cross sections perpendicular to a fixed diameter of the base are squares. (Hint: Center the base at the origin.)

11) As viewed from above, a swimming pool has the shape of the ellipse

$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

The cross sections of the pool perpendicular to the ground and parallel to the y axis are squares. If the units are feet, determine the volume V of the pool.

Solutions

6) π

7) a) 30π

   b) π

8) $\frac{19}{4}\pi$

9) 250π

10) $\frac{16}{3}$

11) $\frac{32{,}000}{3}$ cubic-feet

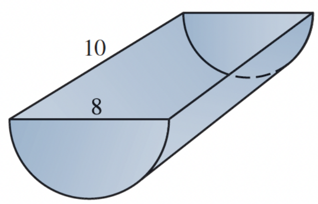
Work (6.4)

12) A 10-pound bag of groceries is to be carried up a flight of stairs 8 feet tall. Find the work W done on the bag.

13) When a certain spring is expanded 10 centimeters from its natural position and held fixed, the force necessary to hold it is 4×10^6 dynes. Find the work W required to stretch the spring an additional 10 centimeters.

14) If 6×10^7 ergs of work are required to compress a spring from its natural length of 10 centimeters to a length of 5 centimeters, find the work W necessary to stretch the spring from its natural length to a length of 12 centimeters.

15) Suppose a large gasoline tank has the shape of a half cylinder 8 feet in diameter and 10 feet long . If the tank is full, set-up the integral to find the work W necessary to pump all the gasoline to the top of the tank. Do NOT evaluate the integral. Assume the gasoline weighs 42 pounds per cubic foot.



16) A tank has the shape of the surface generated by revolving the parabolic segment $y = \frac{1}{2}x^2$ for $0 \leq x \leq 4$ about the y axis. If the tank is full of a fluid weighing 80 pounds per cubic foot, set-up the integral to find the work W required to pump the contents of the tank to a level 4 feet above the top of the tank. Do NOT evaluate the integral. (Hint: Integrate along the y axis.)

Solutions

12) 80 foot-pounds
13) 6×10^7 ergs
14) 9.6×10^6 ergs
15) $W = \int_{-4}^{0} 42(0-x)20\sqrt{16-x^2}\,dx$
16) $W = \int_{0}^{8} 80(12-y)2\pi y\,dy$

Moments and Center of Gravity (6.5)

17) Calculate the center of gravity of the region R between the graphs of f and g on the given interval.

    a) $f(x) = 2x - 1, g(x) = x - 2; [2, 5]$
    b) $f(x) = 2 - x, g(x) = -(2 - x); [0, 2]$

18) Use the symmetry of the region R to determine the center of gravity of R. R is bounded by the hexagon with vertices at $(0, 0), (0, 6)$, $(1, 1), (1, 5), (-1, 1)$, and $(-1, 5)$.

Solutions

17) a) $(\bar{x}, \bar{y}) = \left(\frac{11}{3}, 4\right)$

    b) $(\bar{x}, \bar{y}) = \left(\frac{2}{3}, 0\right)$

18) $(\bar{x}, \bar{y}) = (0, 3)$

Length/Parametrization of a Curve (6.2, 6.7, 6.8)

19) First find an equation relating x and y, when possible. Then sketch the curve C whose parametric equations are given. If C is a circle, indicate the direction $P(t)$ moves as t increases.

   a) $x = 2\cos t$ and $y = 2\sin t$ for $0 \leq t \leq \pi/2$
   b) $x = 3\sin t$ and $y = 3\cos t$ for $-\pi/2 \leq t \leq \pi/2$
   c) $x = -2 + 3t$ and $y = 2 - 3t$ for all t
   d) $x = e^{-t}$ and $y = e^{3t}$ for all t

20) Find the length L of the graph of the given function.

   a) $f(x) = 2x + 3$ for $1 \leq x \leq 5$
   b) $f(x) = 2/3x^{3/2}$ for $1 < x < 4$
   c) $f(x) = \dfrac{1}{8}x^2 - \ln x$ for $1 \leq x \leq 3$

21) Find the length L of the curve described parametrically by $x = 1 - t^2$ and $y = 1 + t^3$ for $0 \leq t \leq 1$.

22) Suppose an object moves in an elliptical orbit parametrized by

$$x = 2\cos t \text{ and } y = 3\sin t$$

where t represents time.

   a) Find a formula for the velocity.
   b) Determine the points on the ellipse at which the object is moving fastest and the points at which it is moving slowest.

Solutions

19) a) $x^2 + y^2 = 4$ for $0 \leq x \leq 2, 0 \leq y \leq 2$
Counter-clockwise

b) $x^2 + y^2 = 9$ for $-3 \leq x \leq 3, 0 \leq y \leq 3$
Clockwise

c) $x = -y$

d) $y = 1/x^3$ for $x > 0$

20) a) $4\sqrt{5}$

b) $\frac{2}{3}(5\sqrt{5} - 2\sqrt{2})$

c) $1 + ln(3)$

21) $\frac{1}{27}(13\sqrt{13} - 8)$

22) a) $\sqrt{4\sin^2 t + 9\cos^2 t}$

b) The velocity is minimum at the points $(0, 3)$ and $(0, -3)$ on the ellipse, and the velocity is maximum at the points $(2, 0)$ and $(-2, 0)$ on the ellipse.

Inverse Functions (7.1); Exponential (7.2) and Logarithmic (7.3) Functions

23) Determine whether the given function has an inverse. If an inverse exists, give the domain and range of the inverse.

   a) $f(t) = \sqrt{1 - t^2}$

   b) $f(x) = x - \sin x$

24) Find the largest intervals on which $f = x^2 - 3x + 2$ has an inverse.

25) Find the largest interval containing $x = 2$ on which $f(x) = \dfrac{1}{1 + x^2}$ has an inverse.

26) Calculate $\left(f^{-1}\right)'(c)$. (Hint: Find a such that $f(a) = c$ by inspection.)

   a) $f(x) = x^3 + 7; c = 6$

   b) $f(x) = x + \sqrt{x}; c = 2$

27) Find the derivative of $y = e^{2x} \ln x$.

28) $\displaystyle\int e^{\sqrt{2}x+3}\, dx$

Solutions

23) a) $f(-t) = f(t)$ for $-1 \le t \le 1$, so f does not have an inverse.

b) $f'(x) = 1 - \cos x \ge 0$, and $f'(x) > 0$ for $x \ne 2n\pi$ for any integer n. Thus f is increasing on any bounded interval and hence on $(-\infty, \infty)$. Therefore f has an inverse. Domain of f^{-1} : $(-\infty, \infty)$; range of f^{-1} : $(-\infty, \infty)$.

24) $f'(x) = 2x - 3$. Since $f'(x) > 0$ for $x > \frac{3}{2}, f$ has an inverse on $\left[\frac{3}{2}, \infty\right)$. Since $f'(x) < 0$ for $x < \frac{3}{2}, f$ has an inverse on $\left(-\infty, \frac{3}{2}\right]$.

25) $f'(x) = -2x/\left(1 + x^2\right)^2$. Since $f'(x) > 0$ for $x < 0, f$ has an inverse on $(-\infty, 0]$, and since $f'(x) < 0$ for $x > 0, f$ has an inverse on $[0, \infty)$. The interval that contains $x = 2$ is $[0, \infty)$.

26) a) $\frac{1}{3}$

b) $\frac{2}{3}$

27) $e^{2x}\left(2\ln x + \frac{1}{x}\right)$

28) $\frac{1}{\sqrt{2}}e^{\sqrt{2}x+3} + C$