





Sustaining Active Learning in Undergraduate Precalculus: Results and Challenges of a Course Redesign

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ABSTRACT

Addressed toward course coordinators and departmental leaders interested in redesigning undergraduate mathematics courses with a focus on active learning, this report provides an update on progress in redesigning the Precalculus course offered by the Department of Mathematics at the University of Maryland, College Park, following initial indicators of success. We begin by discussing our team's challenges due to the COVID-19 pandemic and our attempts to navigate these challenges. We then provide results related to student achievement, self-efficacy, and STEM intentions, in addition to addressing the remaining goals of the redesign.

KEYWORDS

Active learning; precalculus; undergraduate mathematics; course redesign; COVID-19 pandemic; instructional practices; student achievement; self-efficacy; STEM intentions

1. INTRODUCTION AND CONTEXT OF THE PROJECT

Many colleges and universities across the United States continue to adopt active-learning practices in STEM courses, focusing on student engagement and peer-to-peer discussions centered around complex problems [6,18,25]. Such practices are implemented to address, among other trends, a declining number of STEM degrees conferred [9,12,19,23]. Born out of the NSF-funded Student Engagement in Mathematics through an Institutional Network for Active Learning, or SEMINAL Network [4], our team of instructors, graduate students, and campus leaders at the University of Maryland, College Park (UMCP) sought to redesign the Precalculus course offered by the Department of Mathematics with a focus on active learning.

As shared in our 2020 report [17], an initial analysis of student achievement provided the evidence of downward trends in DFW rates due to our redesign efforts. Following our 2020 report, three goals remained:

- (1) Develop a uniform means of analyzing trends in student success using data from the final exam
- (2) Further explore potential relationships between student outcomes and the implementation of active learning in Precalculus

- (3) Disaggregate student-outcome data following the disproportionate number of traditionally underrepresented students in STEM who take Precalculus at UMCP.

We begin by highlighting our team's challenges in working towards these goals due to the COVID-19 pandemic and our attempts to navigate these challenges. We then present new results since our 2020 report demonstrating a relationship between the course redesign and student outcomes.

2. NAVIGATING A PANDEMIC

One of the greatest challenges our Precalculus redesign team faced was navigating the COVID-19 pandemic. All courses at UMCP switched to online instruction roughly halfway through the spring-2020 semester and were entirely online during the fall-2020 and spring-2021 semesters. Precalculus resumed in-person instruction in the fall-2021 semester. Unfortunately, almost all active-learning resources created prior to 2020 were designed with in-person interactions in mind. These resources included, for example, group assignments and explorations conducted during in-person recitation sections. In the following sections, we discuss the challenge of adapting the redesign components to be compatible with virtual instruction. Additionally, we summarize our team's difficulties in analyzing student outcomes of interest during the pandemic.

2.1. Loss of Active-Learning Space

A major loss of the redesign during the pandemic was the use of a dedicated active-learning space. Collaboration amongst students and responding to instructor questions can be daunting from both an instructor and student perspective. Upon reviewing student and instructor data from the fall-2021 semester, during which instruction returned to being in person, only a handful of students shared that they were willing to answer questions with their peers listening in a large-lecture hall. Furthermore, interviewed students from the fall-2021 semester often shared that collaboration occurred more in the smaller recitation sections where they engaged with group problems and explorations.

Even so, part of our success in implementing active learning, including in the large lecture, can be attributed to our team's access to the lecture rooms in the Edward St. John Learning and Teaching Center on campus. Aside from the semesters of virtual instruction, all Precalculus courses have been taught in the Center since its opening in 2017. This space offers the benefit of swiveling chairs for student-to-student collaboration, whiteboards encircling the hall, and multiple projector and television screens for easy viewing of an instructor's slides and notes captured by a document camera (see our 2020 report for more on the positive impacts of the Center on the Precalculus redesign).

During the pandemic, any collaboration between students encouraged during the large-lecture meetings of the course was dependent on students' willingness to unmute themselves on a call with more than 150 classmates. The use of chat threads and breakout rooms, resulting in many groups the instructor had to switch between during a Zoom call, relied on students' willingness to speak with other students in their group, many of whom had their video cameras turned off. Even with the instructor and a designated helper checking the Zoom chat, there was no easy way to efficiently and realistically replicate the Center's design from the ground up with active learning in mind.

Dr. Rosca shared that using the active-learning materials during virtual instruction "was much more problematic" [27]. While Dr. Rosca and the teaching assistants attempted to utilize the group assignments and exploration problems by having students submit their work electronically, checking for student engagement in an online environment was difficult. Dr. Rosca's difficulties during the pandemic were shared by students and college math instructors across the world who experienced the challenge of engagement and the ability to monitor learning during virtual instruction [7,26].

2.2. A Step Back in Analysis

The pandemic also presented several challenges in working towards our first lingering goal of analyzing student success. As a part of the redesign, we aimed to develop a uniform means of year-to-year analysis of student achievement across iterations and versions of the course taught by different instructors with (even if slightly) different syllabi and grading schemes. Following suit with researchers' use of final exams and other summative assessments as measures of achievement [10,24], we chose to use the common final exam administered across all sections of Precalculus as the source of data for such an analysis.

Precalculus instructors at UMCP have historically given a traditional paper-pencil final exam administered in a 2-hour window at a date and time designated by the Department of Mathematics. This common final exam is typically made up of multi-part questions that focus on having students solve equations, describe characteristics of functions, find the equation of a graph, and engage in other skills emphasized in the course. Succinctly, our team aimed to document possible connections between particular instructional practices and student achievement on the final exam, among other student outcomes. Virtual instruction caused such an analysis to be postponed for several reasons.

To begin, introducing virtual instruction raised several concerns around academic integrity. More specifically, there was no way of knowing what unapproved assistance a student may have received in taking the final exam online that would have resulted in an inaccurate representation of student understanding. The fluctuating format of the final exam also raised questions about the validity of, for instance, a comparison of student performance on the final exam during the pandemic with that of a student taking the final exam on paper during a semester in

which in-person instruction resumed. Indeed, the format of the Precalculus final exam changed slightly from semester-to-semester as instructors learned from the difficulties of electronic administration of the final exam during the pandemic.

As an example of such a format, the spring-2020 final exam was given via WebAssign (an online system used to generate and manage assignments) and used a set of pooled problems so that students were given tasks of the same type but with different numbers. Students then submitted their written work for each question, including typed responses when appropriate, to a corresponding final exam assignment in Canvas (the learning management system used at UMCP). Such varying formats introduced a layer of complexity in conducting a uniform analysis of student performance on the final exam over time. For example, using WebAssign meant a portion of a student's grade was drawn from multiple choice responses, a format not historically used on the Precalculus final exam.

We underscore that an analysis of instructional practices and student final exam grades during the pandemic would not have been representative of the redesign efforts used during in-person instruction in preceding years. Succinctly, our goal to explore potential connections between final exam grades and the active-learning practices emphasized during the redesign misaligned with the mode of instruction during the pandemic. Additionally, such an analysis would have been particularly complex when accounting for the switch from in-person to virtual instruction part way through the spring semester of 2020.

Lastly, our team deemed the data collection required for conducting an analysis using the final exam as an additional burden to our redesign team, Precalculus instructors, and students during an already stressful transition to online learning. Such an analysis would have called for students and instructors to participate in our data collection during a time when week-to-week assignments and preparation already required a significant amount of pivoting. In summary, the pandemic introduced several new variables and logistical roadblocks, making our plans to draw connections between Precalculus students' final exam grades and our redesign efforts extremely difficult to implement.

3. ADDRESSING OUR GOALS

Upon returning to in-person instruction during the fall semester of 2021, our team sought to resume a review of student outcomes, beginning with an analysis of students' final exam grades. As highlighted in our 2020 report, three versions of Precalculus are offered at UMCP, including Main Precalculus, which follows a traditional large-lecture and recitation-section structure. Main Precalculus has been the most significant focus of our redesign efforts following the second author's involvement as the course coordinator and large-lecture instructor. Thus we focus the results presented below, organized by the remaining goals of the redesign, on the implementation of active learning in Main Precalculus, hereon referred to as Precalculus. We begin by addressing our first remaining goal, namely, to explore

potential connections between the redesign efforts and student final exam grades, among other student outcomes.

3.1. Goal #1: Uniform Analysis of Student Outcomes

As a part of the larger SEMINAL project, our team has routinely administered the Postsecondary Instructional Practices Survey for Mathematics [3], or PIPS-M, most notably used in the Progress through Calculus Project [22]. The survey allowed instructors and students to comment on practices implemented within the Precalculus large lecture and recitation sections. In addition, our team has routinely interviewed Precalculus instructors and students to expound on their experiences in the course. From this consideration of the unique perspectives of instructors and students, our team sought to explore potential connections between the instructional practices, as perceived by instructors and students, and students' final exam grades.

Considering the negative trends referenced at the outset of this paper related to the number of STEM degrees conferred, we also sought to measure the potential relationship between the perceived instructional practices and students' sense of self-efficacy and STEM intentions. The PIPS-M already includes items related to students' STEM intentions. However, there are no existing items on the PIPS-M that measure self-efficacy. Thus, as an addendum to the Qualtrics survey that included the PIPS-M items, we included items from the Precalculus Self-efficacy Instrument (PSEI) developed by Carter [8] as a valid and reliable tool to measure self-efficacy within the context of courses covering precalculus content. More specifically, the outcome generated from the PSEI provides a numeric value representative of a student's self-efficacy as it pertains to precalculus content, following the context-specific definition of self-efficacy put forth by Bandura [5]. Students' PIPS-M responses were then regressed onto this self-efficacy outcome. Such an analysis theoretically aligns with literature that highlights the connections between the instructional practices used in post-secondary mathematics classrooms and students' confidence in mathematics [2]. To retain validity and reliability of the PIPS-M and PSEI, the items from each respective survey were kept as separate sections in the larger Qualtrics survey given to students.

The findings presented below stem from data¹ collected during the fall-2021 semester. The first author invited participants by visiting the course coordinator's (the second author) large lecture and each of the discussion sections to share about the study. All survey administration and interviews occurred outside of class time. Participating students and instructors received Amazon gift cards in amounts commensurate with their participation (i.e., completing the survey and participating in interviews).

The first author reviewed participating student and instructor responses to the PIPS-M, in addition to transcripts of instructor and student interviews, to consider those PIPS-M items most relevant to the Precalculus course. Student responses to

¹ IRB project 1810279-2.

the relevant PIPS-M items were then used to run a factor analysis for the large lecture and recitation sections separately. The resulting factors defined particular dimensions of teaching that encompassed the instructional practices implemented in the classroom as perceived by students and instructors. Nesting students by the teaching assistant that taught their recitation section, hierarchical models were used to capture potential relationships between students' survey responses surrounding instructional practices and students' final exam grades, self-efficacy, and STEM intentions (see [16] for a more detailed report of the methodology).

Out of the 249 students taking Precalculus during the fall-2021 semester, a total of 181 (approximately 73%) students completed the Qualtrics survey containing the PIPS-M and PSEI items, while the large-lecture instructor (the second author) and four of the five teaching assistants completed the instructor version of the survey. Additionally, all participating instructors, as well as two students of each of the participating teaching assistants, were interviewed. As an overview of the student outcomes of interest, 146 of the 181 students (roughly 81%) who participated in the study earned a grade of 60% or above on the final exam, with a class average of approximately 68% ($SD = 17.56$). When asked during the last three weeks of the semester if they had declared or intended to declare a STEM major, approximately 72% said yes. The average self-efficacy score for participants, before standardization, was 1439.06 ($SD = 398.53$) out of a maximum score of 2000.

3.2. Goal #2: Analysis of Student Outcomes

The greatest motivating factor for redesigning Precalculus was a need to address the large number of students unsuccessfully completing the course. Thus we present Table 1 to succinctly summarize correlations between the instructional practices used in Precalculus and students' final exam grades, in particular. The practices (i.e., PIPS-M items) were coded to account for their alignment with the tenets of active learning. The practices were then summarized by the dimension of teaching resulting from the factor analysis of students' responses. Each dimension was disaggregated by the context of students' responses with respect to either the Precalculus large lecture or discussion sections. While other items were included in our analysis (e.g., attendance), we forgo a discussion of these variables to remain within the scope of this paper (see [16] for more details). To summarize, the dimensions of teaching (i.e., factors) included the following:

- Variation in Instruction – practices related to having students engage in exploring different solution pathways and alternative representations of problems.
- Student-to-Student Collaboration – practices related to having students discuss their difficulties with their classmates.
- Instructor-to-Student engagement – practices related to whole-class sharing and having students respond to questions posed by the instructor.

A hierarchical regression involving these dimensions of teaching revealed significant relationships between students' perceptions of certain practices nested

Table 1. Estimates of fixed effects on student final exam grades.

Parameter	Estimate	Std. Error	t	Sig.
Intercept	72.93	2.50	29.14	<0.001
Large Lecture – Variation in Instruction	5.52***	1.41	3.92	<0.001
Large Lecture – Student-to-Student Collaboration	0.49	1.33	0.37	0.713
Large Lecture – Instructor-to-Student Engagement	−6.73***	1.51	−4.46	<0.001
Discussion – Variation in Instruction	−0.59	1.53	−0.39	0.699
Discussion – Student-to-Student Collaboration	−1.22	1.53	−0.80	0.427
Discussion – Instructor-to-Student Engagement	4.47*	1.72	2.60	0.010

Note: The reference group is a White-male student who neither missed at least one class per week in the large lecture, 4–6 discussion sections, nor at least one discussion section a week. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$.

under a given dimension and students' final exam grades. For example, having students engage with practices related to considering different solution pathways and representations (see the variable *variation in instruction* in Table 1) during the large lecture was significantly predictive of an increase in students' final exam grades. A similar positive relationship was true when considering teaching assistants' posing and answering of questions in the discussion sections (see *instructor-to-student engagement*). In addition to the results presented in Table 1, when considering students' sense of self-efficacy, a significant positive relationship also existed between instructional practices related to *variation in instruction* in the large lecture and *instructor-to-student engagement* in the discussion sections. No significant relationship was found between the student-perceived instructional practices implemented in either the large lecture or discussion sections and students' STEM intentions (a finding we return to in Section 4.3 highlighting the benefit of collaboration across departments and programs).

It is worth noting that the implementation of instructional practices related to *instructor-to-student engagement* in the large lecture was negatively correlated with students' final exam grades. Indeed, evidence from student interviews suggested only some students were willing to engage with the instructor (e.g., answering questions) in front of their peers in a large lecture hall. Thus, it may be that some student-centered approaches to learning may be most effective only in smaller classroom settings (see [16] for a larger discussion on these interpretations).

In our 2020 report, we shared evidence of a downward trend in the Precalculus DFW rate that was below the historical 30–50% rate for the course. As seen in Figure 1, the mean DFW rate between the fall-2018 and fall-2020 semesters was roughly 26%. Unfortunately, beginning in the spring-2021 semester, DFW rates sharply increased following the onset of the pandemic. The mean DFW rate increased to approximately 41% between the spring-2021 and fall-2022 semesters, with a significant peak of around 58% in the spring of 2022. Considering the spring-2022 cohort of Precalculus students would have included students repeating the course following the fall-2021 semester (a common occurrence for any spring semester) and students taking their first year of college following an entire year (i.e., between fall-2020 and spring-2021) of online instruction during their senior year of high school, this dramatic increase in the DFW rate was not surprising to our team.

Even when accounting for semesters during and following the pandemic, the average DFW rate since the fall-2018 semester is approximately 31%, sitting at

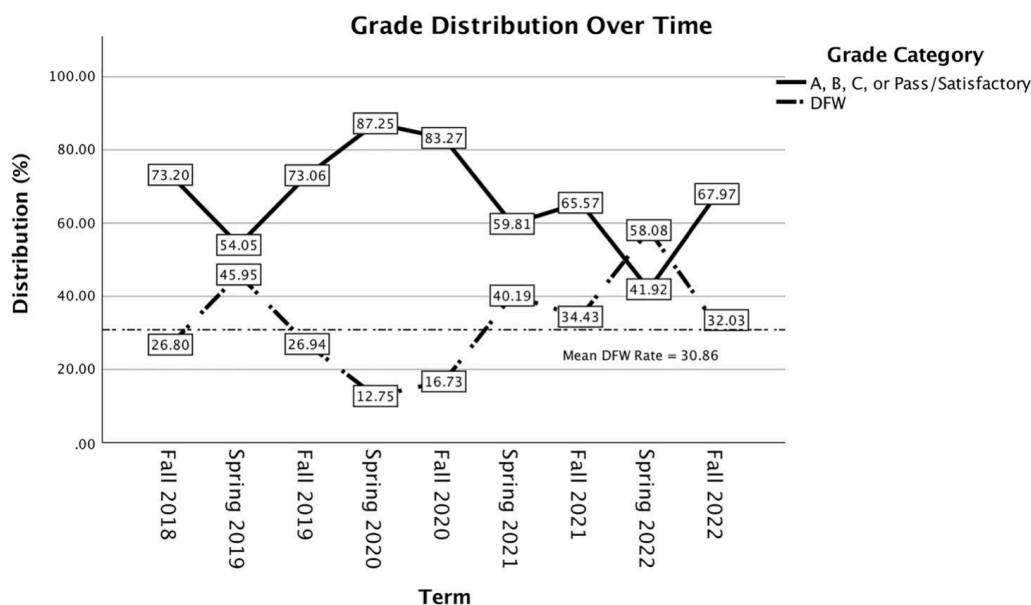


Figure 1. Grade distribution in redesigned Precalculus sections over time. Following the onset of the pandemic, the University of Maryland allowed students to opt for earning a Pass/Satisfactory grade in their courses instead of a specific letter grade beginning in the spring-2020 semester. This option was last offered in the spring-2021 semester.

the lower end of the historical average for the course and aligning with findings reported in our 2020 report. Additionally, in measuring the permeation of active learning throughout Precalculus, instructor and student interviews and survey responses suggested active learning was generally a normative part of the recitation sections and sometimes a part of the lecture meetings during the fall of 2021. These findings, combined with the evidence cited above suggesting a mostly positive-predictive relationship between student outcomes and instructional practices centered around active learning, suggest our redesign efforts continue to support the general population of Precalculus students.

Our team sees the historically low mean DFW rate as just the beginning. We continue to work on implementing redesign efforts across the different components of the course to consistently minimize the DFW rate across semesters. This work is critical when considering the typically higher-than-average DFW rates of spring cohorts of Precalculus that include transfer students and students repeating the class from the fall. In the description below, we discuss our use of on-campus resources to help with this work as we move forward with the redesign.

3.3. Goal #3: Findings from Disaggregated Data

Even when accounting for an increase in DFW rates following the pandemic, the above findings are encouraging when considering the academic difficulties students have historically had in Precalculus at UMCP and the declining national trends mentioned in the introduction of this report. However, there is certainly still work to be done. Further analysis of the final exam data collected during the fall semester of

2021 revealed a significant difference between the final exam grades of White-male students and students traditionally underrepresented in STEM, including students identifying as African American, Hispanic/Latinx, and multiracial. Considering Precalculus has historically consisted of a disproportionate number of students traditionally underrepresented in STEM, this finding is alarming.

One explanation for this achievement gap may be that the Precalculus students traditionally underrepresented in STEM may not be receiving the necessary resources *outside* of the classroom that research has found to support this group of students in their academic achievement, including peer networks and mentoring relationships with faculty [11,15]. This explanation is substantiated when considering the above trends suggesting a positive relationship between the implementation of active learning in Precalculus and the general improvement of the academic achievement of students on the final exam. In other words, empirical evidence recorded here and in our 2020 report suggests that active learning in the large lecture and recitation sections is a beneficial means of instruction for Precalculus students, in general. Thus, factors *outside* of the class may be important to consider in supporting students traditionally underrepresented in STEM, in particular, who are taking Precalculus. Still, we see the redesign of Precalculus as progressing in our aim to support a diverse group of learners in their STEM achievement and STEM intentions. While not a statistically significant result, when asked at the end of the fall-2021 semester about their STEM intentions, African American students taking Precalculus were approximately 2.5 times more likely to intend on majoring in a STEM field when compared to White-male students.

4. MOVING FORWARD

Roughly 6 years into the redesign of Precalculus, our team continues to revisit and add to our repository of active-learning resources. However, with many of these resources having been improved over several semesters of implementation, we have shifted our focus to refining the above student-outcome analysis. We also continue to explore ways to onboard new team members to sustain our redesign efforts. Finally, we look for collaborative opportunities across departments and campus programs that can support the redesign work. We comment on each of these foci below.

4.1. Refinement of Student-Outcome Analysis

A primary goal of the redesign moving forward is refining the initial analysis described above in exploring potential connections between instructional practices and student outcomes. One methodological dilemma we faced in our analysis of student final exam grades is the need for more alignment between our instructional team's use of active-learning resources and the questions asked on the final exam. For instance, the final exam rarely has students answer a question using two different approaches as a means of students checking their work. Additionally, the final

exam does not have students create their own problems. While such active-learning problems are emphasized in the course (see Problem 3 in Appendix A for a sample problem given in recitation sections), these problems are difficult to include on a final exam due to issues of academic integrity, time constraints, and the complexities of grading student-generated problems. Even so, our team continues to explore ways of better aligning the tenets of active learning infused in the Precalculus activities and assignments with questions on the final exam. In turn, we hope to more fully understand the potential benefit of active learning on students' conceptual understanding that may be less apparent on instructor-written exams focused on more traditional problem-solving [14].

4.2. Sustaining Change and Sharing Insights

While we have continued to share our work with other institutions via conferences and presentations, we aim to better engage the instructors and researchers within the Department of Mathematics at UMCP who were not a part of the original redesign team. Part of the struggle to onboard new team members has been due to our team's focus on the time-consuming tasks of the redesign itself (i.e., developing active-learning resources and conducting the above analysis), especially when faced with the difficult circumstances of the pandemic. However, considering the positive findings as a result of the redesign, it is crucial that we share what we have learned with our colleagues and administration in the Department and bring new team members on board to sustain the changes made to Precalculus.

Initial steps have been taken to share our work with others in the Department of Mathematics. For example, the first author's dissertation centered around the Precalculus redesign. As a graduate student of the Center for Mathematics Education (CfME) housed in the College of Education at UMCP, the first author invited the Chair of the Department of Mathematics to serve as the Dean's Representative on his dissertation committee. Additionally, our team has periodically contacted other graduate students in the CfME at UMCP with research interests in mathematics teaching at the collegiate level. Still, we aim to involve additional partners in the Department of Mathematics.

In reflecting on this aim, our team has clear evidence that opportunities exist to share more about the redesign with others in the Department of Mathematics. In particular, the Associate Chair for Undergraduate Studies in the Department, Dr. Larry Washington, has recently taken notice of the successes of the redesign. When asked to comment on any noticeable differences related to Precalculus since the implementation of the course redesign, Dr. Washington offered the following reflection regarding a significant drop in student complaints about Precalculus following the fall-2022 and spring-2023 semesters.

In any lower-level course, I always expect that there are some disgruntled students. However, much to my amazement, I received no complaints about Math 115 [Precalculus] in either semester this year. Something must be going well! [28]

Such comments demonstrate that key leaders in the Department are taking note of the positive effects of the redesign and indicate that other opportunities may exist to share successes from the redesign with others in the Department. From these opportunities, our current team may be able to onboard new members and receive valuable resources from the Department to sustain the changes made to Precalculus moving forward.

4.3. Collaboration Across Departments and Programs

Our team continues to look for other course-redesign opportunities within the Department of Mathematics, as well as opportunities to support students outside of their scheduled course times. Such opportunities do exist, in no small part due to the Learning and Teaching Center's support of instructors' implementation of active learning across departments at UMCP. Moreover, we underscore that other students in the Center for Mathematics Education (CfME) have researched the teaching practices of instructors and teaching assistants within the Department of Mathematics [13,20], in addition to teaching several math courses offered by the Department. Furthermore, the first author recently completed the pilot of a winter workshop offered by the Department of Mathematics to review prerequisite skills and preview Precalculus content as an additional support to students getting ready to take the course in the spring. We hope that relationships between these different resources and the Department of Mathematics continue to grow so that all parties involved can offer their expertise to support undergraduate students taking mathematics courses at UMCP. From this work, our team can continue to grow and engage in meaningful conversations and movements around active learning in mathematics departments across the United States.

In addition to connections with CfME and the Learning and Teaching Center, other existing resources on campus can support students taking Precalculus, particularly those traditionally underrepresented in STEM. The Academic Achievement Programs (AAP) at UMCP is one such resource. In particular, AAP offers a Summer Transition Program (STP) that is designed to support first-generation and low-income students by providing, among other services, tutoring and study-skill classes [1]. Not only do several students typically take Precalculus immediately following STP, but the Department of Mathematics has also worked with AAP in past summers to offer summer credit to students in the Program. Additionally, the first author has served as a math instructor for nine sessions of STP. Thus, we see AAP as a valuable resource for the Department of Mathematics to support Precalculus students.

We envision a potential collaboration between programs like AAP and Precalculus instructors to ensure alignment in what is reviewed during tutoring sessions and what is currently being covered in the Precalculus lectures. Considering there was no significant relationship between the instructional practices within Precalculus classrooms and students' STEM intentions during the fall of 2021, such collaborations may be an essential part of the redesign moving forward. Through

building relationships between Precalculus instructors and programs such as AAP, our redesign team may better provide the support outside of the classroom that empirical evidence suggests is vital to helping first-year and traditionally underrepresented students in STEM succeed in and persist in STEM courses, such as the availability of faculty support [29] and mentoring [21].

5. CONCLUDING REMARKS

As we look to grow our team, we highlight the positive impact of various professional networks in helping us navigate our redesign. Most notably, the SEMINAL-supported Networked Improvement Community (NIC) has been the most influential source of encouragement and ideas on instruction and analysis throughout the redesign. Monthly meetings during the pandemic with other universities conducting similar redesign work provided invaluable insight into resources and best practices to use during virtual instruction, as well as moral support during a challenging time that required everyone to adapt. Additionally, presentations from and conversations with other SEMINAL universities at mathematics and mathematics education conferences (e.g., JMM and RUME) continue to challenge our team to consider new ways in which our redesign can be sustained.

Since joining the SEMINAL Network in 2017, our redesign team at UMCP has seen encouraging results that can be tied to our implementation of active learning in Precalculus. In our 2020 report, we saw downward trends in DFW rates of Precalculus and captured anecdotal evidence from instructors and students suggesting the positive impact active learning had on their experience and success in the course. As highlighted in the current report, positive trends in student outcomes have continued following continued implementation of the redesign. Moreover, these positive results have begun to catch the attention of influential leaders in the Department of Mathematics.

There is certainly still work to be done, particularly in supporting Precalculus students traditionally underrepresented in STEM, as well as collaborating with others to sustain the changes made to Precalculus. Furthermore, some findings presented here suggest student-centered practices may not always result in improved student outcomes, particularly in a large lecture. From this perspective, student-centered practices are not a perfect ‘fix’ for addressing concerns around student outcomes in undergraduate math classrooms. Still, our team is proud of the progress we have made in redesigning Precalculus with a focus on active learning. We see the redesign of Precalculus as having sparked a larger movement to reflect on the state of undergraduate mathematics courses at the University of Maryland and make necessary changes to better support students’ learning experiences.

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APPENDIX A: SAMPLE RECITATION PROBLEM

In the following problems, you will practice working with exponential functions.

- (1) Sketch the graph of the function $g(x) = 10 - 2 \cdot 5^{-x}$. If you did not obtain $g(0) = 8$, please review the order of operations (PEMDAS).
- (2) Suppose a function goes through the points $(-2, 28)$ and $(0, 4)$, and has a horizontal asymptote of $y = 3$. Find a function of the form $f(x) = A \cdot b^x + c$ with these characteristics.
 - 2.1 Based on the horizontal asymptote, find c .
 - 2.2 Use the value for c found above, and the fact that the graph goes through the point $(0, 4)$, that is $f(0) = 4$, to find A .
 - 2.3 Rewrite the function $f(x)$, substituting A and c , as found above. Now, use the fact that the graph goes through the point $(-2, 28)$, that is $f(-2) = \underline{\hspace{2cm}}$, to find b .
 - 2.4 Write the final form of your function.

Think of a transformed exponential function. Draw its graph (but do not write its equation) and write down clues similar to the ones in Problem 2 above. See if someone else from your group can find your function.

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REFERENCES

- [1] AAP's Purpose, Goals, and Mission. *n.d.* *Academic Achievement Programs*. Accessed 13 April 2023.
- [2] American Mathematical Association of Two-Year Colleges. 2006. Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College (R. Blair, Ed.). AMATYC. <http://beyondcrossroads.matyc.org/doc/PDFs/BCAll.pdf>.
- [3] Apkarian, N., W. M. Smith, K. Vroom, M. Voigt, J. Gehrtz, and PtC Project Team, & SEMINAL Project Team. 2019. X-PIPS-M Survey Suite. <https://www.maa.org/sites/default/files/XPIPSM%20Summary%20Document.pdf>.
- [4] Association of Public and Land-Grant Universities. 2018, March 22. SEMINAL. SEMINAL. <http://www.aplu.org/projects-and-initiatives/stem-education/seminal>. Accessed 21 Nov, Maryland 2018.
- [5] Bandura, A. 1977. Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*. 84(2): 191–215. <https://www.ncbi.nlm.nih.gov/pubmed/847061>.
- [6] Boaler, J. and J. G. Greeno. 2000. Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple Perspectives on Mathematics Teaching and Learning* (Vol. 1, pp. 171–200). Westport, CT: Ablex Publishing.
- [7] Bonsangue, M. V. and J. E. Clinkenbeard. 2021. A comparison of American student and faculty experiences in mathematics courses during the COVID-19 pandemic. *International Journal of Educational Research Open*. 2(100075): 100075.

- [8] Carter, N. 2022. *Measuring pre-calculus self-efficacy, grit, and achievement in online flipped university pre-calculus courses* [Unpublished doctoral dissertation]. (M. Peters, Ed.). University of Houston, Clear Lake.
- [9] Chen, X. 2013. *STEM attrition: College students' paths into and out of STEM fields*. National Center for Education Statistics. <https://nces.ed.gov/pubs2014/2014001rev.pdf>.
- [10] Duffin, L. C., H. B. Keith, M. I. Rudloff, and J. D. Cribbs. 2020. The effects of instructional approach and social support on college algebra students' motivation and achievement: Classroom climate matters. *International Journal of Research in Undergraduate Mathematics Education*. 6(1): 90–112.
- [11] Ellington, R. M. and R. Frederick. 2010. Black high achieving undergraduate mathematics majors discuss success and persistence in mathematics. *The Negro Educational Review*. 61: 61–84.
- [12] Fayer, S., A. Lacey, and A. Watson. 2017. *BLS Spotlight on Statistics: STEM Occupations-Past, Present, and Future*. Washington, D.C.: {U.S. Department of Labor, Bureau of Labor Statistics}. <https://www.bls.gov/spotlight/2017/science-technology-engineering-and-mathematics-stem-occupations-past-present-and-future/pdf/science-technology-engineering-and-mathematics-stem-occupations-past-present-and-future.pdf>.
- [13] Fleming, E. 2017. Positioning in an upper-level undergraduate mathematics course [Unpublished doctoral dissertation]. (D. Chazan, Ed.). University of Maryland, College Park.
- [14] Freeman, S., S. L. Eddy, M. McDonough, M. K. Smith, N. Okoroafor, H. Jordt, and M. P. Wenderoth. 2014. Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*. 111(23): 8410–8415.
- [15] Griffin, K. A., D. Pérez, A. P. Holmes, and C. E. Mayo. 2010. Investing in the future: The importance of faculty mentoring in the development of students of color in STEM. *New Directions for Institutional Research*. 2010(148): 95–103.
- [16] Gruber, S. 2022. Exploring the classroom norms of an undergraduate precalculus course and their relationship with students' self-efficacy, achievement, and STEM intentions: A convergent mixed-methods study [Unpublished doctoral dissertation]. (A. Brantlinger, Ph.D.). University of Maryland, College Park.
- [17] Gruber, S., R. I. Rosca, D. Chazan, E. Fleming, S. Balady, C. VanNetta, and K. A. Okoudjou. 2020. Active learning in an undergraduate precalculus course: Insights from a course redesign. *PRIMUS*. 31(3-5): 358–370.
- [18] Hmelo-Silver, C. E. 2004. Problem-based learning: What and how do students learn? *Educational Psychology Review*. 16(3): 235–266.
- [19] Langen, A. v. and H. Dekkers. 2005. Cross-national differences in participating in tertiary science, technology, engineering and mathematics education. *Comparative Education*. 41(3): 329–350.
- [20] Lue, K. 2022. Navigating the multiple roles of mathematics graduate teaching assistants in pursuit of racial equity, access, and justice. In S. Clem (Ed.), *Exploring how we Teach: Lived Experiences, Lessons, and Research About Graduate Instructors by Graduate Instructors* (pp. 78–96). Logan, UT: Utah State University. <https://uen.pressbooks.pub/exploreteaching/chapter/navigating-the-multiple-roles-of-mathematics-graduate-teaching-assistants-in-pursuit-of-racial-equity-access-and-justice/>. Accessed 26 May 2023
- [21] MacPhee, D., S. Farro, and S. S. Canetto. 2013. Academic self-efficacy and performance of underrepresented STEM majors: Gender, ethnic, and social class patterns. *Analyses of Social Issues and Public Policy*. 13(1): 347–369.
- [22] Mathematical Association of America. 2015-2019. *Progress through calculus* (No. DUE I-USE #1430540). National Science Foundation. <https://www.maa.org/programs-and-communities/curriculum%20resources/progress-through-calculus>.

- [23] National Center for Education Statistics. 2017. *Data point: Beginning college students who change their majors within 3 years of enrollment*. US Department of Education. <https://files.eric.ed.gov/fulltext/ED578434.pdf>.
- [24] Peters, M. L. 2013. Examining the relationships among classroom climate, self-efficacy, and achievement in undergraduate mathematics: A multi-level analysis. *International Journal of Science and Mathematics Education*. 11(2): 459–480.
- [25] Prince, M. 2004. Does active learning work? A review of the research. *Journal of Engineering Education*. 93(3): 223–231.
- [26] Radmehr, F. and S. Goodchild. 2022. Switching to fully online teaching and learning of mathematics: The case of Norwegian mathematics lecturers and university students during the COVID-19 pandemic. *International Journal of Research in Undergraduate Mathematics Education*. 8(3): 581–611.
- [27] Rosca, R. 2023, April 11. Interview with Sean Gruber.
- [28] Washington, L. C. 2023, June 27. Email with Sean Gruber and Raluca Rosca.
- [29] Xu, Y. J. 2018. The experience and persistence of college students in STEM majors. *Journal of College Student Retention: Research, Theory & Practice*. 19(4): 413–432.

BIOGRAPHICAL SKETCHES

Sean Gruber is a lecturer in the Department of Mathematics at the University of Maryland, College Park (UMCP). In addition to having taught Precalculus and other courses offered by the Department, Gruber is also involved in designing bridge programs for the Precalculus and College Algebra courses offered by the Department. Gruber currently serves as Co-Chair for the Math Success Initiatives Team at UMCP, a group charged by the Provost with the task of addressing a set of initiatives to support student success in undergraduate mathematics courses. Additionally, Gruber is engaged in research related to classroom practices and tools that may provide avenues for students to give instructors feedback during the semester within large-lecture classes. Before working at UMCP, Gruber served as a visiting assistant professor in the Department of Mathematics at Johns Hopkins University. While at Johns Hopkins, Gruber served as a lead instructor for Calculus I and Calculus II for biological and social science majors. In addition to bringing an STEM-education lens to the entry-level courses at Johns Hopkins, Gruber assisted in developing logistic regression models to analyze trends in students' math placement exam results and their performance in their first college math class at Johns Hopkins. His research interests include exploring how the social norms and instructional practices that make up a college math classroom affect students' self-efficacy, achievement, and STEM intentions/persistence. More specifically, Gruber is interested in researching and implementing student-centered instructional practices (i.e., active learning) to improve college students' views of and experiences in undergraduate mathematics courses.

Raluca Rosca is a lecturer in mathematics at the University of Maryland, College Park (UMCP), and is in her seventh year as chairperson of the Precalculus lecture course offered by the Mathematics Department. She has over 15 years of teaching experience at the college level, and since coming to UMCP has taught Precalculus in both the large lecture format and the single-contact summer or Freshman Connection format for twenty semesters. As the daughter of two first-generation college students and the administrator of an outreach program to middle schools, she is also very interested in using Precalculus as a stepping stone to success in college mathematics, not just in terms of content, but also in terms of creating the social network and the personal skills, habits and dispositions that empowers today's diverse students of Precalculus.